Package ‘highOrderPortfolios’

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Title Design of High-Order Portfolios Including Skewness and Kurtosis

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Description The classical Markowitz's mean-variance portfolio formulation ignores heavy tails and skewness. High-order portfolios use higher order moments to better characterize the return distribution. Different formulations and fast algorithms are proposed for high-order portfolios based on the mean, variance, skewness, and kurtosis.
The package is based on the papers:
R. Zhou and D. P. Palomar (2021). ``Solving High-Order Portfolios via Successive Convex Approximation Algorithms.'' <doi:10.48550/arXiv.2008.00863>.
X. Wang, R. Zhou, J. Ying, and D. P. Palomar (2022). ``Efficient and Scalable High-Order Portfolios Design via Parametric Skew-t Distribution.'' <doi:10.48550/arXiv.2206.02412>.

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URL https://github.com/dppalomar/highOrderPortfolios,
https://www.danielppalomar.com

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`highOrderPortfolios-package`
highOrderPortfolios: Design of High-Order Portfolios via Mean, Variance, Skewness, and Kurtosis

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Description

The classical Markowitz's mean-variance portfolio formulation ignores heavy tails and skewness. High-order portfolios use higher order moments to better characterize the return distribution. Different formulations and fast algorithms are proposed for high-order portfolios based on the mean, variance, skewness, and kurtosis.

Functions

`design_MVSK_portfolio_via_sample_moments()`, `design_MVSK_portfolio_via_skew_t()`, and `design_MVSKtilting_portfolio_via_sample_moments()`

Help

For a quick help see the README file: GitHub-README.

Author(s)

Rui Zhou, Xiwen Wang, and Daniel P. Palomar

References

R. Zhou and D. P. Palomar, "Solving High-Order Portfolios via Successive Convex Approximation Algorithms," in *IEEE Transactions on Signal Processing*, vol. 69, pp. 892-904, 2021. <https://doi.org/10.1109/TSP.2021.305

X. Wang, R. Zhou, J. Ying, and D. P. Palomar, "Efficient and Scalable High-Order Portfolios Design via Parametric Skew-t Distribution," Available in arXiv, 2022. <https://arxiv.org/pdf/2206.02412.pdf>.

`design_MVSKtilting_portfolio_via_sample_moments`

Design high-order portfolio by tilting a given portfolio to the MVSK efficient frontier

---

Description

Design high-order portfolio by tilting a given portfolio to the MVSK efficient frontier (i.e., mean, variance, skewness, and kurtosis):

```
minimize     - delta
             m1(w) >= m1(w0) + delta*d1
             m2(w) <= m2(w0) - delta*d2
             m3(w) >= m3(w0) + delta*d3
             m4(w) <= m4(w0) - delta*d4
             (w-w0)'Sigma(w-w0) <= kappa^2
subject to   ||w||_1 <= leverage, sum(w) == 1.
```

Usage

```
design_MVSKtilting_portfolio_via_sample_moments(
  d = rep(1, 4),
  X_moments,
  w_init = rep(1/length(X_moments$mu), length(X_moments$mu)),
  w0 = w_init,
  w0_moments = NULL,
  leverage = 1,
  kappa = 0,
  method = c("Q-MVSKT", "L-MVSKT"),
  tau_w = 1e-05,
  tau_delta = 1e-05,
  gamma = 1,
  zeta = 1e-08,
  maxiter = 100,
  ftol = 1e-05,
  wtol = 1e-05,
  theta = 0.5,
  stopval = -Inf
)
```

Arguments

`d`	Numerical vector of length 4 indicating the weights of first four moments.
`X_moments`	List of moment parameters, see `estimate_sample_moments()`.
`w_init`	Numerical vector indicating the initial value of portfolio weights.
`w0`	Numerical vector indicating the reference portfolio vector.
`w0_moments`	Numerical vector indicating the reference moments.
`leverage`	Number (>= 1) indicating the leverage of portfolio.
`kappa`	Number indicating the maximum tracking error volatility.
`method`	String indicating the algorithm method, must be one of: "Q-MVSK", "MM", "DC".
`tau_w`	Number (>= 0) guaranteeing the strong convexity of approximating function.
`tau_delta`	Number (>= 0) guaranteeing the strong convexity of approximating function.
`gamma`	Number (0 < gamma <= 1) indicating the initial value of gamma.
`zeta`	Number (0 < zeta < 1) indicating the diminishing paramater of gamma.
`maxiter`	Positive integer setting the maximum iteration.
`ftol`	Positive number setting the convergence criterion of function objective.
`wtol`	Positive number setting the convergence criterion of portfolio weights.
`theta`	Number (0 < theta < 1) setting the combination coefficient when enlarge feasible set.
`stopval`	Number setting the stop value of objective.

Value

A list containing the following elements:

`w`	Optimal portfolio vector.
`delta`	Maximum tilting distance of the optimal portfolio.
`cpu_time_vs_iterations`	Time usage over iterations.
`objfun_vs_iterations`	Objective function over iterations.
`iterations`	Iterations index.
`moments`	Moments of portfolio return at optimal portfolio weights.
`improvement`	The relative improvement of moments of designed portfolio w.r.t. the reference portfolio.

Author(s)

Rui Zhou and Daniel P. Palomar

References

R. Zhou and D. P. Palomar, "Solving High-Order Portfolios via Successive Convex Approximation Algorithms," in *IEEE Transactions on Signal Processing*, vol. 69, pp. 892-904, 2021. <doi:10.1109/TSP.2021.3051369>.

Examples

```
library(highOrderPortfolios)
data(X50)

# estimate moments
X_moments <- estimate_sample_moments(X50[, 1:10])

# decide problem setting
w0 <- rep(1/10, 10)
w0_moments <- eval_portfolio_moments(w0, X_moments)
d <- abs(w0_moments)
kappa <- 0.3 * sqrt(w0 %*% X_moments$Sgm %*% w0)

# portfolio optimization
sol <- design_MVSKtilting_portfolio_via_sample_moments(d, X_moments, w_init = w0, w0 = w0,
                                                        w0_moments = w0_moments, kappa = kappa)
```

---

design_MVSK_portfolio_via_sample_moments

Design high-order portfolio based on weighted linear combination of first four moments

---

Description

Design high-order portfolio based on weighted linear combination of first four moments (i.e., mean, variance, skewness, and kurtosis):

```
minimize     - lmd1*(w'*mu) + lmd2*(w'*Sigma*w)
             - lmd3*(w'*Phi*w*w) + lmd4*(w'*Psi*w*w*w)
subject to   ||w||_1 <= leverage, sum(w) == 1.
```

Usage

```
design_MVSK_portfolio_via_sample_moments(
  lmd = rep(1, 4),
  X_moments,
  w_init = rep(1/length(X_moments$mu), length(X_moments$mu)),
  leverage = 1,
  method = c("Q-MVSK", "MM", "DC"),
  tau_w = 0,
  gamma = 1,
  zeta = 1e-08,
  maxiter = 100,
  ftol = 1e-05,
  wtol = 1e-04,
  stopval = -Inf
)
```

Arguments

lmd	Numerical vector of length 4 indicating the weights of first four moments.
X_moments	List of moment parameters, see estimate_sample_moments().
w_init	Numerical vector indicating the initial value of portfolio weights.
leverage	Number (>= 1) indicating the leverage of portfolio.
method	String indicating the algorithm method, must be one of: "Q-MVSK", "MM", "DC".
tau_w	Number (>= 0) guaranteeing the strong convexity of approximating function.
gamma	Number (0 < gamma <= 1) indicating the initial value of gamma.
zeta	Number (0 < zeta < 1) indicating the diminishing paramater of gamma.
maxiter	Positive integer setting the maximum iteration.
ftol	Positive number setting the convergence criterion of function objective.
wtol	Positive number setting the convergence criterion of portfolio weights.
stopval	Number setting the stop value of objective.

Value

A list containing the following elements:

w	Optimal portfolio vector.
cpu_time_vs_iterations	Time usage over iterations.
objfun_vs_iterations	Objective function over iterations.
iterations	Iterations index.
convergence	Boolean flag to indicate whether or not the optimization converged.
moments	Moments of portfolio return at optimal portfolio weights.

Author(s)

Rui Zhou and Daniel P. Palomar

References

R. Zhou and D. P. Palomar, "Solving High-Order Portfolios via Successive Convex Approximation Algorithms," in *IEEE Transactions on Signal Processing*, vol. 69, pp. 892-904, 2021. <doi:10.1109/TSP.2021.3051369>.

X. Wang, R. Zhou, J. Ying, and D. P. Palomar, "Efficient and Scalable High-Order Portfolios Design via Parametric Skew-t Distribution," Available in arXiv, 2022. <https://arxiv.org/pdf/2206.02412v1.pdf>.

Examples

```
library(highOrderPortfolios)
data(X50)

# estimate moments
X_moments <- estimate_sample_moments(X50[, 1:10])

# decide moment weights
xi <- 10
lmd <- c(1, xi/2, xi*(xi+1)/6, xi*(xi+1)*(xi+2)/24)

# portfolio optimization
sol <- design_MVSK_portfolio_via_sample_moments(lmd, X_moments)
```

`design_MVSK_portfolio_via_skew_t`

Design MVSK portfolio without shorting based on the parameters of generalized hyperbolic skew-t distribution

---

Description

Design MVSK portfolio without shorting based on the parameters of generalized hyperbolic skew-t distribution:

```
minimize     - lambda1*phi1(w) + lambda2*phi2(w)
             - lambda3*phi3(w) + lambda4*phi4(w)
subject to   w>=0, sum(w) == 1.
```

Usage

```
design_MVSK_portfolio_via_skew_t(
  lambda,
  X_skew_t_params,
  w_init = rep(1/length(X_skew_t_params$mu), length(X_skew_t_params$mu)),
  method = c("L-MVSK", "DC", "Q-MVSK", "SQUAREM", "RFPA", "PGD"),
  gamma = 1,
  zeta = 1e-08,
  tau_w = 0,
  beta = 0.5,
  tau = 1e+05,
  initial_eta = 5,
  maxiter = 1000,
  ftol = 1e-06,
  wtol = 1e-06,
  stopval = -Inf
)
```

Arguments

lambda	Numerical vector of length 4 indicating the weights of first four moments.
X_skew_t_params	List of fitted parameters, including location vector, skewness vector, scatter matrix, and the degree of freedom, see estimate_skew_t().
w_init	Numerical vector indicating the initial value of portfolio weights.
method	String indicating the algorithm method, must be one of: "L-MVSK", "DC", "Q-MVSK", "SQUAREM", "RFPA", "PGD".
gamma	Number (0 < gamma <= 1) indicating the initial value of gamma for the Q-MVSK method.
zeta	Number (0 < zeta < 1) indicating the diminishing parameter of gamma for the Q-MVSK method.
tau_w	Number (>= 0) guaranteeing the strong convexity of approximating function.
beta	Number (0 < beta < 1) decreasing the step size of the projected gradient methods.
tau	Number (tau > 0) hyper-parameters for the fixed-point acceleration.
initial_eta	Initial eta for projected gradient methods
maxiter	Positive integer setting the maximum iteration.
ftol	Positive number setting the convergence criterion of function objective.
wtol	Positive number setting the convergence criterion of portfolio weights.
stopval	Number setting the stop value of objective.

Value

A list containing the following elements:

w	Optimal portfolio vector.
cpu_time_vs_iterations	Time usage over iterations.
objfun_vs_iterations	Objective function over iterations.
iterations	Iterations index.
convergence	Boolean flag to indicate whether or not the optimization converged.
moments	Moments of portfolio return at optimal portfolio weights.

Author(s)

Xiwen Wang, Rui Zhou and Daniel P. Palomar

References

X. Wang, R. Zhou, J. Ying, and D. P. Palomar, "Efficient and Scalable High-Order Portfolios Design via Parametric Skew-t Distribution," Available in arXiv, 2022. <https://arxiv.org/pdf/2206.02412.pdf>.

Examples

```
library(highOrderPortfolios)
data(X50)

# estimate skew t distribution
X_skew_t_params <- estimate_skew_t(X50)

# decide moment weights
xi <- 10
lambda <- c(1, 4, 10, 20)

# portfolio optimization
sol <- design_MVSK_portfolio_via_skew_t(lambda, X_skew_t_params, method = "RFPA", tau = 10)
```

---

estimate_sample_moments *Estimate first four moment parameters of multivariate observations*

Description

Estimate first four moments of multivariate observations, namely, mean vector, covariance matrix, coskewness matrix, and cokurtosis matrix.

Usage

```
estimate_sample_moments(X, adjust_magnitude = FALSE)
```

Arguments

`X` Data matrix.

`adjust_magnitude` Boolean indicating whether to adjust the order of magnitude of parameters. Note: this is specially designed for the function `design_MVSKtilting_portfolio_via_sample_moment`

Value

A list containing the following elements:

`mu`	Mean vector.
`Sgm`	Covariance matrix.
`Phi_mat`	Co-skewness matrix.
`Psi_mat`	Co-kurtosis matrix.
`Phi`	Co-skewness matrix in vector form (collecting only the unique elements).
`Psi`	Co-kurtosis matrix in vector form (collecting only the unique elements).
`Phi_shred`	Partition on `Phi` (see reference).
`Psi_shred`	Partition on `Psi` (see reference).

Author(s)

Rui Zhou and Daniel P. Palomar

References

R. Zhou and D. P. Palomar, "Solving High-Order Portfolios via Successive Convex Approximation Algorithms," in *IEEE Transactions on Signal Processing*, vol. 69, pp. 892-904, 2021. <doi:10.1109/TSP.2021.3051369>.

Examples

```
library(highOrderPortfolios)
data(X50)

X_moments <- estimate_sample_moments(X50[, 1:10])
```

---

estimate_skew_t — *Estimate the parameters of skew-t distribution from multivariate observations*

Description

Using the package fitHeavyTail to estimate the parameters of ghMST distribution from multivariate observations, namely, location vector (mu), skewness vector (gamma), scatter matrix (scatter), degree of freedom (nu), parameters a, and the Cholesky decomposition of the scatter matrix (chol_Sigma).

Usage

```
estimate_skew_t(
  X,
  initial = NULL,
  nu_lb = 9,
  max_iter = 100,
  ptol = 0.001,
  ftol = Inf,
  PXEM = TRUE,
  return_iterates = FALSE,
  verbose = FALSE
)
```

Arguments

`X`	Data matrix containing the multivariate time series (each column is one time series).
`initial`	List of initial values of the parameters for the iterative estimation method. Possible elements include:

- nu: default is 4,
- mu: default is the data sample mean,
- gamma: default is the sample skewness vector,
- scatter: default follows from the scaled sample covariance matrix,

nu_lb	Minimum value for the degree of freedom to maintain the existence of high-order moments (default is 9).
max_iter	Integer indicating the maximum number of iterations for the iterative estimation method (default is 100).
ptol	Positive number indicating the relative tolerance for the change of the variables to determine convergence of the iterative method (default is 1e-3).
ftol	Positive number indicating the relative tolerance for the change of the log-likelihood value to determine convergence of the iterative method (default is Inf, so it is not active). Note that using this argument might have a computational cost as a convergence criterion due to the computation of the log-likelihood (especially when X is high-dimensional).
PXEM	Logical value indicating whether to use the parameter expansion (PX) EM method to accelerating the convergence.
return_iterates	Logical value indicating whether to record the values of the parameters (and possibly the log-likelihood if ftol < Inf) at each iteration (default is FALSE).
verbose	Logical value indicating whether to allow the function to print messages (default is FALSE).

Value

A list containing the following elements:

mu	Location vector estimate (not the mean).
gamma	Skewness vector estimate.
scatter	Scatter matrix estimate.
nu	Degrees of freedom estimate.
chol_Sigma	Choleski decomposition of the Scatter matrix estimate.
a	A list of coefficients useful for later computation

Author(s)

Xiwen Wang, Rui Zhou, and Daniel P. Palomar

References

Aas, Kjersti and Ingrid Hobæk Haff. "The generalized hyperbolic skew student'st-distribution," Journal of financial econometrics, pp. 275-309, 2006.

Examples

```
library(highOrderPortfolios)
data("X50")
X_skew_t_params <- estimate_skew_t(X50)
```

`eval_portfolio_moments` *Evaluate first four moments of a given portfolio*

Description

Evaluate first four moments of a given portfolio's return, namely, mean, variance, skewness, and kurtosis.

Usage

```
eval_portfolio_moments(w, X_statistics)
```

Arguments

`w`	Numerical vector with portfolio weights.
`X_statistics`	Argument characterizing the constituents assets. Either the sample parameters as obtained by function `estimate_sample_moments()` or the multivariate skew t parameters as obtained by function `estimate_skew_t()`.

Value

Four moments of the given portfolio.

Author(s)

Rui Zhou, Xiwen Wang, and Daniel P. Palomar

References

R. Zhou and D. P. Palomar, "Solving High-Order Portfolios via Successive Convex Approximation Algorithms," in *IEEE Transactions on Signal Processing*, vol. 69, pp. 892-904, 2021. <doi:10.1109/TSP.2021.3051369>.

X. Wang, R. Zhou, J. Ying, and D. P. Palomar, "Efficient and Scalable High-Order Portfolios Design via Parametric Skew-t Distribution," Available in arXiv, 2022. <https://arxiv.org/pdf/2206.02412v1.pdf>.

Examples

```
library(highOrderPortfolios)
data(X50)

# nonparametric case
X_moments <- estimate_sample_moments(X50[, 1:10])
w_moments <- eval_portfolio_moments(w = rep(1/10, 10), X_statistics = X_moments)

# parametric case (based on the multivariate skew t distribution)
X_skew_t_params <- estimate_skew_t(X50[, 1:10])
w_moments <- eval_portfolio_moments(w = rep(1/10, 10), X_statistics = X_skew_t_params)
```

X100 *Synthetic 500x100 matrix dataset*

Description

Synthetic 500x100 matrix dataset containing 500 realizations of 100 variables.

Usage

```
data(X100)
```

Format

An object of class `xts` (inherits from `zoo`) with 500 rows and 100 columns.

X200 *Synthetic 1000x200 matrix dataset*

Description

Synthetic 1000x200 matrix dataset containing 1000 realizations of 200 variables.

Usage

```
data(X200)
```

Format

An object of class `xts` (inherits from `zoo`) with 1000 rows and 100 columns.

X50 *Synthetic 250x50 matrix dataset*

Description

Synthetic 250x50 matrix dataset containing 250 realizations of 50 variables.

Usage

```
data(X50)
```

Format

An object of class `matrix` (inherits from `array`) with 250 rows and 50 columns.

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